Resistance Approximation In High Harmonic Frequencies

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Summary: The problem of consideration of the resistance in calculation of non-sinusoidal regimes in electrical networks is considered in article. It is shown the necessity of taking in account the skin effect in harmonic frequencies. Using general electromagnetic field theory in condition medium and Bessel's function allow to obtain expressions of resistance approximations in high harmonics frequencies for conductors of round cross-sectional areas.

Key words:
skin effect, high harmonics, propagation, induction, conductor, frequency

1. INTRODUTION

The calculation of losses in electrical power networks is impossible without the consideration of resistance variation in high harmonic frequencies. Correct consideration of resistance in calculations of non-sinusoidal conditions is one of the most complex and necessary problems. In most cases the resistance of power supplying components in equivalent circuit are either neglected or are given constant value R_0, equaling the resistance to a direct current. It is only in rare cases that the development of skin effect in harmonic frequencies are considered, using under this condition expression:

$$R_\nu = \sqrt{\nu} R_0 \tag{1}$$

where:
ν — the order of harmonic.

Many authors raise doubt as regards the use of this expression for resistance calculation in harmonic frequencies. Usually, this concerns harmonics of non-high orders. Apart from this, the question arises on which method to calculate the resistance, given a current spectra having at the same time, many frequencies. And really, this is the most frequent situation in practice. Could the superposition principle be applied here or will the resistance in one frequency be dependent on other frequencies in the current spectra?

2. SKIN EFFECT IN PLANE CONDUCTORS

Let's try to answer this question employing the general electromagnetic field theory in condition medium.

It is known that the propagation of electromagnetic wave in conducting medium is characterized by the complex coefficient of propagation:

$$\delta = \alpha + j\beta \tag{2}$$

where:
α — damping factor
β — phase factor which are connected with magnetic μ and electric ε permeabilities and conductivity γ of the conductor and also with the frequency ω by the expression:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\gamma^2}{\varepsilon^2\omega^2}}-1\right)} \tag{3}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\gamma^2}{\varepsilon^2\omega^2}}+1\right)} \tag{4}$$

In a good condition medium:

$$\frac{\gamma}{\varepsilon\omega} >> 1 \tag{5}$$

Therefore:

$$\alpha \approx \beta \approx \sqrt{\frac{\mu\gamma\omega}{2}} \tag{6}$$

Let's look at the propagation of electromagnetic wave in plane conductors (Fig. 10). Since the relationship (5) is justified for conductors, then under any given incident wave angle, the direction of its propagation will be perpendicular to the boundary surface while the electric and magnetic field intensity vectors $\overline{E}$ and $\overline{H}$ will be parallel to its boundary (Fig. 1).

The complex amplitude $\dot{H}$ is related to the amplitude $\dot{E}$ by the relationship:

$$\dot{H} = \frac{\alpha + j\beta}{j\omega\mu}\dot{E} \tag{7}$$

or considering (6):

$$H = \sqrt{\frac{\gamma}{\mu\omega}}E \tag{8}$$

where the vector $\overline{H}$ lags in angle to vector $\overline{E}$ by:

$$\varphi = arctg\frac{\beta}{\alpha} = \frac{\pi}{4} \tag{9}$$

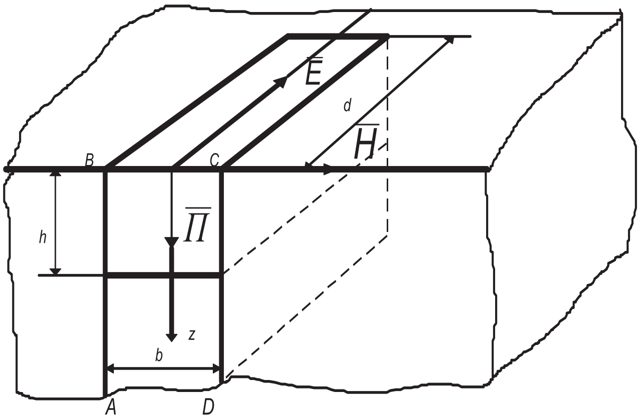

Fig. 1. Plane conductor in electromagnetic field

Let electromagnetic wave comprise of n oscillations of different frequencies $\omega_1, \omega_2 ... \omega_n$. Then the values of electric and magnetic field intensities in z depth will be:

$$E(t) = \sum_{i=1}^{n} E_i e^{-a_i z} \sin(\omega_i t + \beta_i z + \psi_i) \qquad (10)$$

$$H(t) = \sum_{i=1}^{n} H_i e^{-a_i z} \sin\left(\omega_i t + \beta_i z + \psi_i - \frac{\pi}{4}\right) \qquad (11)$$

Since the power dissipated in a volume of a rectangular parallelepiped, the base of which lies on the surface of the conductor (Fig. 1), and the side edges extending to infinity, equals the power entering the conductor from the surrounding dielectric through the base of the parallelepiped. The mean value of this power is equal to the product of the mean value of Posting's vector from the surface of the conductor and the area of the base bd.

Posting's vector can be obtained from the formula's (10), (11), given that $z = 0$:

$$\ddot{i}(t) = \left(\sum_{i=1}^{n} E_i \sin(\omega_i t + \psi_i)\right) \times \left(\sum_{i=1}^{n} H_i \sin\left(\omega_i t + \psi_i - \frac{\pi}{4}\right)\right) \qquad (12)$$

Let's find the mean value of $n(t)$ in a T period. Under T period we shall understand the highest of all the periods of frequencies $\omega_1, \omega_2 ... \omega_n$ and multiples of other periods. Then, considering the orthogonality of trigonometrical functions,

$$\ddot{i}_{mid} = \frac{1}{T}\int_0^T \ddot{i}(t)dt = \frac{\sum_{i=1}^{n} E_i H_i}{2\sqrt{2}} \qquad (13)$$

Referring to the expressions (6) and (8), we have:

$$\ddot{i}_{mid} = \frac{\sum_{i=1}^{n} \beta_i H_i^2}{2\gamma} \qquad (14)$$

where:

$$\beta_i = \sqrt{\frac{\mu\gamma\omega_i}{2}} \qquad (15)$$

The real power, dissipated in the conductor:

$$P = bd\ddot{i}_{mid} = \frac{bd}{2\gamma}\sum_{i=1}^{n} \beta_i H_i^2 \qquad (16)$$

Let's find the current, flowing through the parallelepiped. In accordance with Ampere's circuital law:

$$i = \oint_{abcda} \overline{H}d\overline{l} \qquad (17)$$

Since on the sides ab and cd the $\overline{H}$ vector is perpendicular to $\overline{dl}$ and on the side da, as a result of remoteness at an infinite distance, vector $\overline{H}$ **= 0**, then:

$$\oint_{abcda} \overline{H}d\overline{l} = H(t)b = b\sum_{i=1}^{n} H_i \sin\left(\omega_i t + \psi_i - \frac{\pi}{4}\right) \qquad (18)$$

Rms. values of the current:

$$I = \frac{b}{\sqrt{2}}\sqrt{\sum_{i=1}^{n} H_i^2} \qquad (19)$$

The resistance is calculated as the ratio of real power P to the square of rms. value of the current I^2:

$$R = \frac{P}{I^2} = \frac{d}{b\gamma}\frac{\sum_{i=1}^{n} \beta_i H_i^2}{\sum_{i=1}^{n} H_i^2} = \frac{d}{b}\sqrt{\frac{\mu}{2\gamma}}\frac{\sum_{i=1}^{n} \sqrt{\omega_i} H_i^2}{\sum_{i=1}^{n} H_i^2} \qquad (20)$$

From equation (20) follows the impossibility application of superposition principle and the fact, that the resistance of a conductor in frequency ω_1 is determined by its characteristics which are dependent only on this frequency and do not depend on the availability of electromagnetic wave of other frequencies:

$$R(\omega_i) = \frac{d}{b}\sqrt{\frac{\omega_i \mu}{2\gamma}} \qquad (21)$$

This same treatment could be given to conductors of round cross sectional areas. Under this condition the resistance is expressed using Bessel's function, and also in this case, the superposition principle seems useful.

For realization of time method of analysis of non-sinusoidal condition in electrical networks and also for the evaluation of apparent power values from the point of view

Table 1. The dependence of module A_ν and α_ν on argument k_ν

k_ν	0	1	2	3	4	5	6	7	8	9	10
A_ν	1	1.01	1.18	1.63	2.17	2.68	3.18	3.68	4.18	4.68	5.18
α_ν	0	7.0	24.0	36.0	39.0	40.4	41.1	41.8	42.2	42.6	42.8

Table 2. The dependence of $\nu_{\min}$ on cross-sectional area for copper

s, mm^2	4	6	10	16	25	35	50	70	95
$\nu_{\min}$	139	93	56	35	23	16	11	8	6
s, mm^2	120	150	185	240	300	400	500	550	625
$\nu_{\min}$	5	4	3	3	2	2	2	1	1

of power losses calculation, the introduction of resistance in equivalent circuits is inevitable. On the other hand, transient condition will last for an infinite duration of time which makes it impossible to obtain a solution for the steady-state condition.

3. CONDUCTORS OF ROUND CROSS-SECTIONAL AREAS

Let's examine the approximation of the relating laws of resistance and the internal inductance with the frequency. During skin effect in conductors of round cross-sectional areas with radius a and length l the internal impedance: complex is expressed through Bessel's function of the first kind:

$$Z_\nu = R_\nu + jX_\nu =$$

$$= \frac{\sqrt{\omega_\nu \mu \gamma} l}{2\pi a \gamma} e^{-j\frac{\pi}{4}} \frac{J_0\left(\sqrt{\omega_\nu \mu \gamma} a e^{-j\frac{\pi}{4}}\right)}{J_1\left(\sqrt{\omega_\nu \mu \gamma} a e^{-j\frac{\pi}{4}}\right)} = \qquad (22)$$

$$= R_0 \frac{h_\nu}{2} \frac{J_0(h_\nu)}{J_1(h_\nu)}$$

where:

$$h_\nu = \sqrt{\omega_\nu \mu \gamma} a e^{-j\frac{\pi}{4}}$$

$J_0(h_\nu)$ and $J_1(h_\nu)$ — Bessel's function of the first kind, which are the argument of zero and first order respectively.

If we use some notation:

$$R \frac{h_\nu}{2} \frac{J_0(h_\nu)}{J_1(h_\nu)} = A_\nu e^{j\alpha_\nu} \qquad (23)$$

then the resistance and inductance in the ν-th harmonic frequency are expressed as follows:

$$R_\nu = R_0 A_\nu \cos\alpha_\nu \qquad (24)$$

$$R_\nu = R_0 \frac{A_\nu}{\nu \omega_0} \sin\alpha_\nu \qquad (25)$$

The dependence of module A_ν and α_ν on $k_\nu = \sqrt{\omega_\nu \mu \gamma} a$ is shown in Table 1.

The dependence of R_ν on ν has sufficiently a complex nature, all the same, with sufficient accuracy for practical purposes it can be approximated with the following expressions:

$$R_\nu = \begin{cases} R_0, & \sqrt{\frac{\omega_0 \mu \gamma \nu s}{\pi}} < 2; \\ \left(0.325 + 0.346\sqrt{\frac{\omega_0 \mu \gamma \nu s}{\pi}}\right) R_0, & \sqrt{\frac{\omega_0 \mu \gamma \nu s}{\pi}} \geq 2, \end{cases} \qquad (26)$$

where $s = \pi a^2$ – the cross-sectional area of conductor.

For instance, for copper $\mu = 4\pi \cdot 10^{-7}$ H/m, $\gamma = 5.7 \cdot 10^7$ 1/(Om · m) and when $\omega_0 = 2\pi \cdot 50 = 314$ rad/s

$$\sqrt{\frac{\omega_0 \mu \gamma}{\pi}} = 0.085 \text{ mm}^{-1},$$

while the resistance is expressed as thus:

$$R_\nu = \begin{cases} R_0, & \nu < \nu_{min}; \\ \left(0.325 + 0.0293\sqrt{\nu s}\right) R_0, & \nu \geq \nu_{min}, \end{cases} \qquad (27)$$

where:

s — conductors cross-sectional area, mm^2.

The minimum harmonic number $\nu_{\min}$ for which there is already the need to consider the skin effects expressed with regards to the conductor's cross-sectional area and material:

$$\mu_{min} = \frac{4\pi}{\omega_0 \mu \gamma s} \qquad (28)$$

In table 2, the relationship $\nu_{\min} = f(s)$ are shown for copper.

4. CONCLUSION

From equation (26) it becomes obvious that equation (1) is justified only in the first approximation under the condition that there is a high degree of skin effect not only in the ν-th harmonic frequency but also in the fundamental frequency ω_0, and under R_0, we understand the resistance not to a direct current but to the alternating current with the frequency ω_0, considering the skin effect. In this case, the expression (19) can more correctly be written as follows:

$$R_\nu = \sqrt{\nu} R_1 \tag{29}$$

where:

R_1 — the resistance during a heavy skin effect in ω_0 frequency.

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